PERGAMON

International Journal of Heat and Mass Transfer 43 (2000) 487–491

International Journal of
HEAT and MASS
TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Double-pass heat or mass transfer through a parallel-plate channel with recycle

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Received 17 August 1998; received in revised form 20 April 1999

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1. Introduction

Numerous separation processes and reactor designs with internal or external refluxes at both ends have been developed, such as loop reactors [1,2], air-lift reactors [3,4] and draft-tube bubble columns [5,6], which are broadly used in absorption, fermentation, and polymerization. The recycle ratio and the barrier position were considered in designing heat and mass transfer processes with internal and external refluxes at both ends. The fully developed velocity distribution is assumed and the conjugated Graetz problem is analyzed for uniform wall temperature for this purpose, and numerical solutions and experimental results were carried out in the previous work [7]. An analytical solution is obtained for this type of problem by using orthogonal expansion techniques [8–16], such formulae represent a markedly contribution to the design and analysis of conjugated Graetz problems. It is the purpose of this work to develop the complete theory with orthogonal expansion techniques and investigate the improvement of heat or mass transfer in a double-pass parallel-plate device with external recycle at the ends of conduit.

2. Temperature profiles

Consider an open conduit of thickness W, length L, and width B ($\gg W$) which is divided into two channels with thickness ΔW and $(1-\Delta)W$, respectively, by inserting an impermeable plate of negligible thickness and thermal resistance, as shown in Fig. 1. Before entering the lower channel for a double-pass system, the fluid with volume flow rate V and temperature T_I will mix the fluid exiting from the upper channel with the volume flow rate of recycle RV, which is controlled by means of a conventional pump situated at the beginning of lower channel.

By following the same mathematical treatment performed in the previous works [16], except the type of reflux, the outlet temperature for double-pass devices (θ_F) as well as for single-pass devices ($\theta_{0,F}$) were also obtained in terms of the Graetz number G_Z), eigenvalues (λ_m and $\lambda_{0,m}$), expansion coefficients ($S_{a,m}$, $S_{b,m}$ and $S_{0,m}$), location of impermeable sheet (Δ) and eigenfunctions ($F_{a,m}(\eta_a)$, $F_{b,m}(\eta_b)$ and $F_{0,m}(\eta_0)$). The results are

$$\theta_F = 1-\psi_F = \frac{1}{G_Z}\left[\sum_{m=0}^{\infty}\frac{(1-\mathrm{e}^{-\lambda_m})}{\lambda_m\Delta}S_{a,m}F'_{a,m}(0) + \sum_{m=0}^{\infty}\frac{(1-\mathrm{e}^{-\lambda_m})}{\lambda_m(1-\Delta)}S_{b,m}F'_{b,m}(0)\right] \quad (1)$$

$$\theta_{0,F} = 1-\psi_{0,F} = \frac{1}{G_Z}\sum_{m=0}^{\infty}\left[\frac{(1-\mathrm{e}^{-\lambda_{0,m}})}{\lambda_{0,m}}S_{0,m}F'_{0,m}(0) - \frac{(1-\mathrm{e}^{-\lambda_{0,m}})}{\lambda_{0,m}}S_{0,m}F'_{0,m}(1)\right] \quad (2)$$

In obtaining above results, the velocity profiles in

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PII: S0017-9310(99)00144-1

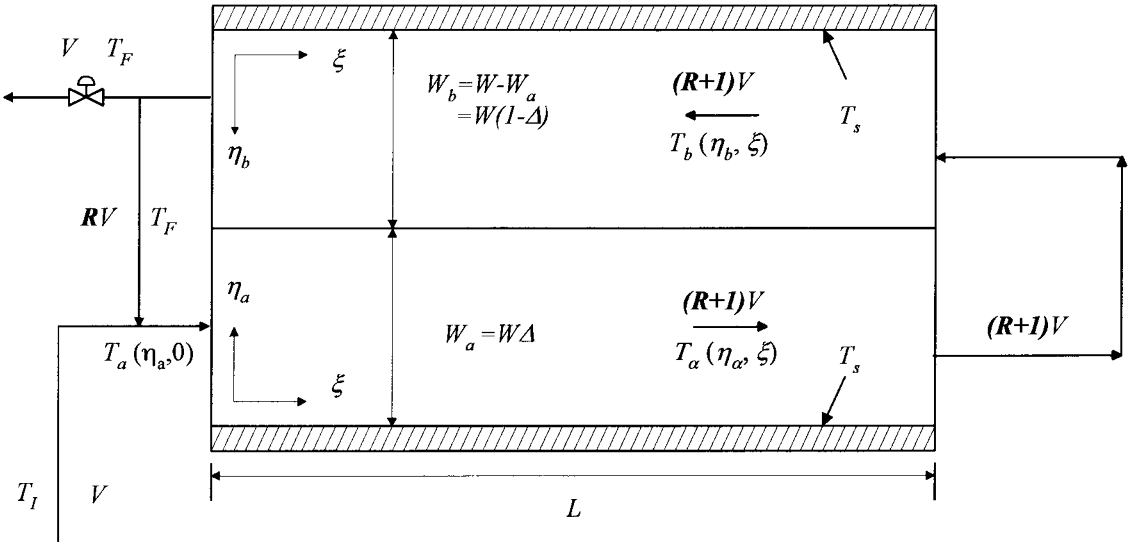

Fig. 1. Double-pass parallel-plate channel with reflux at the outlet.

present double-pass devices, as shown in Eqs. (3) and (4), were modified from those in the previous works since the types of reflux are different while the mathematical analysis is the same.

$$v_a(\eta_a) = \left[\frac{V(R+1)}{W_aB}\right](6\eta_a - 6\eta_a^2), \quad 0 \le \eta_a \le 1 \tag{3}$$

$$v_b(\eta_b) = -\left[\frac{V(R+1)}{W_bB}\right](6\eta_b - 6\eta_b^2), \quad 0 \le \eta_b \le 1 \tag{4}$$

3. Improvement of transfer efficiency

By following the same mathematical treatment performed in the previous work [16], except the type of reflux, the Nusselt number for a double-pass device with recycle may be obtained as follows:

$$\overline{Nu} = \frac{\bar{h}W}{k} = \frac{VW}{2\alpha BL}(1 - \psi_F) = 0.5G_Z\theta_F \tag{5}$$

Similarly, for a single-pass device without recycle

$$\overline{Nu}_0 = \frac{\bar{h}_0W}{k} = \frac{VW}{2\alpha BL}(1 - \psi_{0,F}) = 0.5G_Z\theta_{0,F} \tag{6}$$

The improvement of performance by employing a double-pass device with recycle is

$$I_h = \frac{\overline{Nu} - \overline{Nu}_0}{\overline{Nu}_0} = \frac{\psi_{0,L} - \psi_F}{1 - \psi_{0,F}} = \frac{\theta_F - \theta_{0,F}}{\theta_{0,F}} \tag{7}$$

4. Results and discussion

The calculation methods and procedure are exactly the same as those in the previous work [16], and the results, thus, obtained will be discussed. The changes of mixed inlet temperature $T_a(\eta_a,0)$ and outlet temperature T_F with the reflux ratio R and Graetz number G_Z, as well as the barrier position Δ are in the same tendency as those in the previous works [7,16]. Both $T_a(\eta_a,0)$ and T_F increase with R but decrease as G_Z increases, and the increase of $T_a(\eta_a,0)$ with R is more sensitive for lower R.

The Nusselt numbers, $\overline{Nu}$ and $\overline{Nu}_0$, as well as the transfer coefficiencies, $\bar{h}$ and $\bar{h}_0$, can be calculated from Eqs. (5) and (6), respectively, for a double-pass device with recycle and single-pass device without recycle. Fig. 2 gives the graphical representations of $\overline{Nu}$ and $\overline{Nu}_0$ vs. G_Z. It is seen from this figure that both $\overline{Nu}$ and $\overline{Nu}_0$ increase with G_Z, because $\bar{h}$ and $\bar{h}_0$ will be enhanced as the fluid velocity V increases or the length of flow channel L decreases. Moreover, $\overline{Nu}$ is much larger than $\overline{Nu}_0$, except for very small G_Z.

The comparison of $\overline{Nu}$ obtained in the present and the previous studies of two-pass devices with recycle, is also shown in Fig. 2. One may notice in this figure that the values of $\bar{h}$ in the present device are higher, since the second-flow channel of present device is employed for heating the whole fluid while that of the previous one is provided only for heating the reflux fluid. Further, $\overline{Nu}$ obtained in both devices increase with R, however, the increase in the present device is rather insensitive. This is because the outlet fluid in the

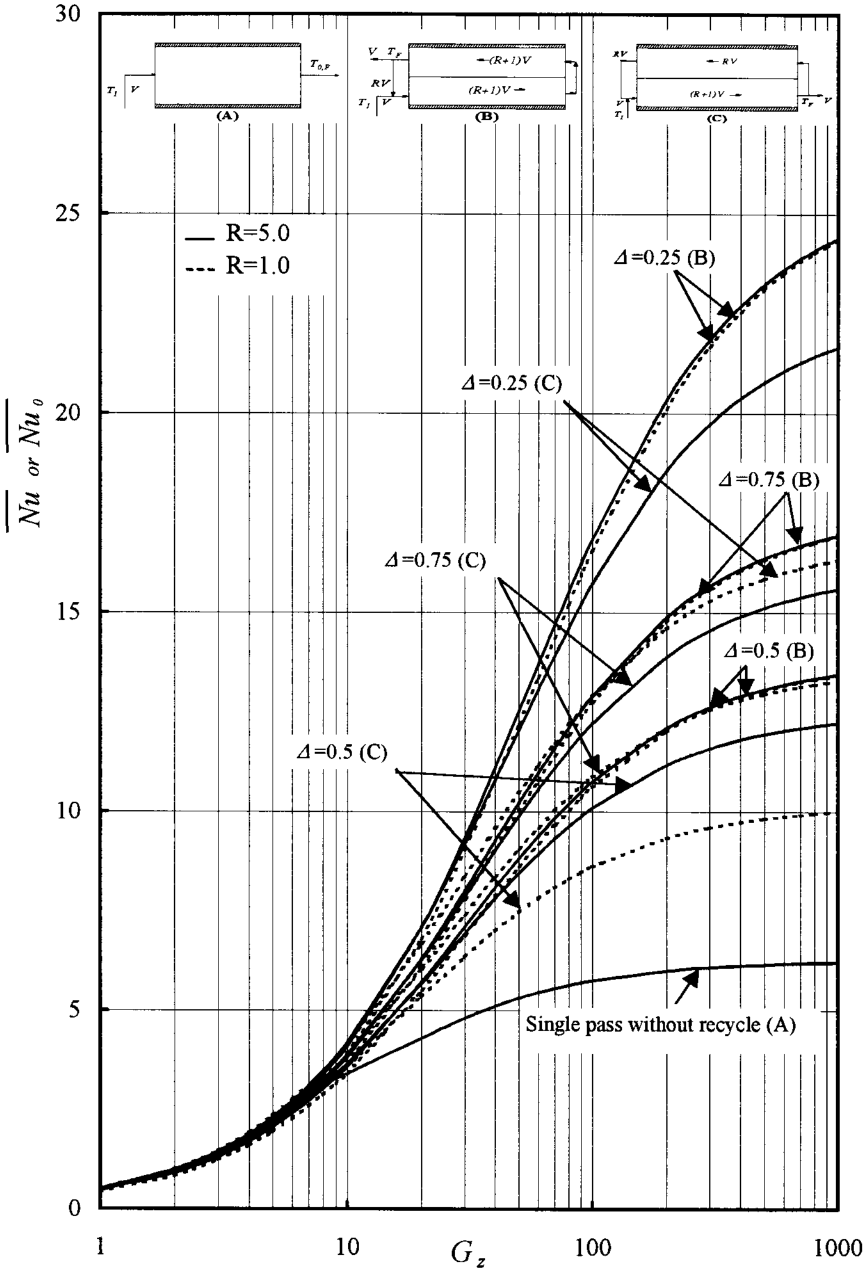

Fig. 2. Nusselt number vs. G_Z with reflux ratio and Δ as parameters.

present device is heated twice in the first- and then the second-flow channels, and the extent of further improvement in transfer efficiency by recycle is rather limited while in the previous device [16], the outlet fluid is heated only in the first-channel and larger amount of reflux fluid (larger R) for preheating the inlet fluid will be more helpful. The effect of Δ on $\overline{Nu}$ is also shown in Fig. 2. $\overline{Nu}$ increases as the value of Δ goes away from 0.5, especially for $\Delta < 0.5$. The reason why $\Delta < 0.5$ is better than $\Delta > 0.5$, for obtaining higher transfer coefficient, is that the heat transfer in the first-flow channel is more effective than that in the second-flow channel due to the higher temperature difference (the driving force of heat transfer). Therefore, decreasing the thickness W_a of the first-flow channel will increase the fluid velocity v_a, leading to improved performance.

Finally, the improvement of performance I_h can be

Table 1
The improvement of the transfer efficiency with reflux ratio and barrier position as parameters

I_h (%)	$R = 1.0$			$R = 2.0$			$R = 5.0$		
	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$
$G_Z = 1$	−12.89	−15.51	−11.93	−9.82	−11.18	−8.35	−5.08	−6.94	−5.34
10	15.07	0.70	8.32	18.03	3.15	6.40	20.50	5.60	12.42
100	187.77	84.77	121.83	190.41	85.99	96.61	192.64	87.21	124.44
1000	292.13	116.11	172.15	293.62	116.40	131.09	294.54	116.59	173.76

obtained from Eq. (7). Some results are given in Table 1. It is found from Table 1 that the improvement in the transfer coefficient of a double-pass device with recycle, based on that of a single-pass device of same dimensions and fluid velocity without recycle, increases with the Graetz number and reflux ratio, as well as with the ratio of thickness Δ going away form 0.5, especially for $\Delta < 0.5$. However, the effect of R on I_h becomes insignificant as G_Z increases. It should be mentioned that the improvement turns to be negative when the Graetz number is sufficiently small, as also shown by the negative signs in Table 1. In this case, a single-pass heat or mass exchanger without recycle is rather recommended to be used than using the double-pass device even with recycle.

If the laminar flow in the flow channels is assumed, the increment of power consumption I_p due to the friction losses ($\ell w_{f,a}$ and $\ell w_{f,b}$ for double pass while $\ell w_{f,0}$ for single pass) in the conduits can readily derived as

$$I_p = \frac{\left(\ell w_{f,a} + \ell w_{f,b}\right) - \left(\ell w_{f,0}\right)}{\ell w_{f,0}}$$

$$= \frac{R+1}{\Delta^3} + \frac{R+1}{(1-\Delta)^3} - 1 \qquad (8)$$

Though the increment of power consumption does not depend on Graetz number, it increases as Δ goes away from 0.5. It is readily obtained from Eq. (8) that I_p increases with reflux ratio as well as with Δ going away from 0.5, and results for I_p are presented in Table 2.

5. Conclusion

The methods for improving the performance in heat (or mass) transfer devices are either the increase of residence time or the production of preheating (or premixing) effect. Actually, the application of recycle to heat (or mass) transfer devices creates two conflicting effects: the desirable preheating effect of the inlet fluid and the undesirable effect of decreasing residence time. At low Graetz number (either small input volume flow rate V or large conduit length L) the residence time is essentially long and should be kept for good performance. In this case, therefore, the preheating effect produced by setting the recycle can not compensate for the decrease of residence time, and hence both the outlet temperature and the Nusselt number in a two-pass device with recycle are lower than those in a single-pass device without recycle, as shown in Table 1. However, the introduction of reflux still has positive effects on the heat transfer for large Graetz number and the outlet temperature as well as transfer coefficient increases with increasing reflux ratio, as also shown in Table 1. This is due to the preheating effect having more influence than the residence-time effect here. Furthermore, both the average outlet temperature and the average Nusselt number change greatly with Graetz number, but change very little with reflux

Table 2
The increment of power consumption with reflux ratio and barrier position as parameters

R	I_p				
	$\Delta = 0.1$ or 0.9	$\Delta = 0.2$ or 0.8	$\Delta = 0.3$ or 0.7	$\Delta = 0.4$ or 0.6	$\Delta = 0.5$
0.5	15	1.89	0.59	0.29	0.23
1.0	20	2.53	0.79	0.40	0.31
2.0	30	3.80	1.19	0.60	0.47
5.0	60	7.61	2.39	1.21	0.95

ratio for large Graetz number. Since the position of the barrier has much influence on the heat transfer behavior, it is presented in Table 1 that I_h increases also as the ratio of the thickness Δ goes away from 0.5, especially for $\Delta < 0.5$.

The present device actually is the extension of another recycle problem in the previous works [7,16], Fig. 2 demonstrates some results obtained in Ref. [16] for comparison. With this comparison, the advantage of present results is evident, especially for lower reflux ratio. It is concluded that with large Graetz number, recycle can enhance heat and mass transfer for the fluid flowing through a parallel-plate channel, by inserting an impermeable plate under double-pass operations.

Acknowledgements

The authors wish to thank the National Science Council of the Republic of China for the financial support.

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